

# Accurate Measurement in the Field of the Earth of the General-Relativistic Precession of the LAGEOS II Pericenter and New Constraints on Non-Newtonian Gravity

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The pericenter shift of a binary system represents a suitable observable to test for possible deviations from the Newtonian inverse-square law in favor of new weak interactions between macroscopic objects. We analyzed 13 years of tracking data of the LAGEOS satellites with GEODYN II software but with no models for general relativity. From the fit of LAGEOS II pericenter residuals we have been able to obtain a 99.8% agreement with the predictions of Einstein's theory. This result may be considered as a 99.8% measurement in the field of the Earth of the combination of the  $\gamma$  and  $\beta$  parameters of general relativity, and it may be used to constrain possible deviations from the inverse-square law in favor of new weak interactions parametrized by a Yukawa-like potential with strength  $\alpha$  and range  $\lambda$ . We obtained  $|\alpha| \lesssim 1 \cdot 10^{-11}$ , a huge improvement at a range of about 1 Earth radius.

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Tests for Newtonian gravity and for a possible violation of the weak equivalence principle are strongly related and represent a powerful approach in order to validate Einstein's theory of general relativity (GR) with respect to alternative theories of gravity and to tune, from the experimental point of view, gravity itself into the realm of quantum physics. Moreover, new long range interactions (NLRIs) may be thought of as the residual of a cosmological primordial scalar field related with the inflationary stage (dilaton scenario) [1]. Twenty-four years ago, the possibility of a fifth force of nature prompted new experimental investigation of possible deviations from the gravitational inverse-square law [2]. In fact, the deviations from the usual  $1/r$  law for the gravitational potential would lead to new weak interactions between macroscopic objects.

Interestingly, these supplementary interactions may be either consistent with Einstein's equivalence principle or not. In this second case, nonmetric phenomena will be produced with tiny, but significant, consequences in the gravitational experiments [3]. The feature of such interactions, which are predicted by several theories, is to produce deviations for separations of masses ranging through several orders of magnitude, starting from the submillimeter level up to the astronomical scale. Among the various techniques useful for the search of this additional physics to the various scales, the accurate measurement of the pericenter shift of binary systems may be used to test for a NLRI with a characteristic range comparable with the system semimajor axis [4].

These very weak NLRI are usually described by means of a Yukawa-like potential with strength  $\alpha$  and range  $\lambda$  and transmitted by a field of very small mass  $\mu = \hbar/\lambda c$ . If  $G_\infty$  represents the gravitational constant,  $M_\oplus$  and  $m_s$

the mass of the primary body and of the satellite,  $r$  their separation,  $c$  the speed of light and  $\hbar$  the reduced Planck constant, we can write:

$$V_{Yuk} = -\alpha \frac{G_\infty M_\oplus}{r} e^{-r/\lambda}, \quad \alpha = \frac{1}{G_\infty} \left( \frac{K_\oplus}{M_\oplus} \frac{K_s}{m_s} \right), \quad (1)$$

where the strength  $\alpha$  depends both on the mass-energy content of the sources and on their coupling strengths,  $K_\oplus$  and  $K_s$ , respectively.

In the weak field and slow motion limit (WFSML) of GR, Einstein's equations reduce to a form quite similar to those of electromagnetism [5]. In Einstein's geometrodynamics and in the frame of a relativistic 3-body problem, where the two primaries are the Sun and the Earth and the test particle is represented by a satellite orbiting the Earth, the main precessions to which the satellite orbit, as a sort of enormous gyroscope, is subject to are commonly known in the literature as: i) Einstein [6], ii) de Sitter [7], and iii) Lense-Thirring (LT) [8] precessions. These precessions may be explained in terms of the effects, on the orbital plane of the satellite, produced by the gravitoelectric and gravitomagnetic fields of the Earth, points i) and iii), and by the effects arising from the coupling between the Earth's motion with the background field of the Sun, point ii). While Einstein's precession is a spin-independent secular effect, the other two precessions are usually interpreted as spin-orbit effects, in particular, as frame-dragging effects, but with some differences: the de Sitter precession is frame-dependent, while the LT one is intrinsically related with the spin of the primary mass, i.e., with its rotation. Therefore, this precession must be related with *intrinsic* gravitomagnetism. We refer to Ciufolini and Wheeler [9] and to Ciufolini [10] for a deeper insight.

This Letter is devoted to showing some of our recent results on the first simultaneous measurements of the cited relativistic precessions in the field of the Earth using the two LAGEOS (LAsER GEODYNamics Satellite) satellites. This work is new with respect to previous ones because we measure all the relativistic secular effects at one time. In particular, we focus on the satellites' pericenter secular advances [11], to which several non-Newtonian theories of gravity are sensitive. We analyzed 13 years of Satellite Laser Ranging (SLR) data of the two LAGEOS satellites using the NASA/GSFC software GEODYN II [12]. This software is dedicated to satellite orbit determination and prediction, geodetic parameters estimation, tracking instruments calibration, and many other applications in the field of space geodesy. The key ingredients of our measurement are (i) a consistent statement of the theory to be tested, (ii) the availability of a good test mass with related high-quality tracking data and (iii) a modelization set for test mass dynamics and tracking.

The first ingredient is far from being trivial: the relativistic equations of motion can be formulated in principle (due to general covariance) in whatsoever coordinate system; however, see Ashby and Bertotti (AB) [13], a suitable choice of this system makes its physical interpretation clearer and simplifies its formulation. In their generalized local inertial frame the main contribution to the test mass dynamics comes from the central body, while third-body effects show up only through tidal terms. The GR acceleration model included in GEODYN II follows the results of Huang *et al.* [14], and represents a generalization of the AB model. The main feature of this model is that their noninertial geocentric frame retains all the merits of the inertial geocentric frame of AB, but it does not rotate with respect to the barycentric reference frame.

The second ingredient is given by the laser ranging data of LAGEOS satellites. The two are almost twins [15]. LAGEOS, launched by NASA (1976), and LAGEOS II, launched by NASA/ASI (1992), have been designed spherical in shape, with high density and small area-to-mass ratio in order to minimize the effects of the subtle and complex nongravitational perturbations [16]. Their radius is just 30 cm and their mass about 407 kg [17]. Their aluminum surface is covered with 426 cube-corner retro-reflectors for laser ranging from dedicated ground stations. The precision of the measurements is mainly related with the pulse width, which is usually  $\approx 1 \cdot 10^{-10}$  s down to  $3 \cdot 10^{-11}$  s for the best laser ranging stations. The SLR data are available through the International Laser Ranging Service (ILRS) [18] in the form of normal points, with a root-mean-square (rms) down to a few mm, that corresponds to an accuracy in the orbit reconstruction at a few cm level, when using the best dynamical models. In our preliminary analyses we have been able to fit the orbit of the satellites at a 1–2 cm (rms) level in range.

Regarding the third ingredient, the models included in

GEODYN II are devoted to describe satellite dynamics, measurement procedure, and reference frame transformations; they include [19]: (i) the geopotential (static and dynamic), (ii) lunisolar and planetary perturbations, (iii) solar radiation pressure and Earth's albedo, (iv) Rubincam and Yarkovsky-Schach effects (which need the satellite spin-axis coordinates), (v) SLR stations coordinates, (vi) ocean loading, (vii) Earth Orientation Parameters and (viii) measurement procedure. Usually, the models implemented in the code include the GR corrections in the parametrized post-Newtonian (PPN) formalism [20].

In the analysis we performed, in order to solve for the relativistic secular precessions, we did not include in our setup such corrections. Moreover, we did not estimate any empirical accelerations as well as the satellites' radiation coefficient  $C_R$ , polar motion and universal time UT1 corrections, in such a way to avoid any possible absorption of physical effects. Finally, in order to avoid the problems related with the spin modeling, also the Rubincam and Yarkovsky-Schach (YS) effects have not been included in our analysis. Concerning the estimated parameters, besides the satellite state-vector, for each 15-day arc we estimated only measurement biases. The long-arc analysis of the orbits of geodetic satellites is a useful way to extract relevant information (i.e., model parameters) concerning the Earth's structure [21]. The physical information is concentrated in the satellite orbital residuals, that must be extracted from the orbital elements determined during the fit [22]. The software fits the tracking data with all its models minimizing the difference between the observed data and the computed ones, using a differential correction procedure. The final residuals are a measure of all the unmodeled effects, such as the GR ones, as well as of the poorly modeled and mismodeled effects and the noise in the tracking data. The relativistic precessions are effective in the orbital elements that define the orbit orientation in space, i.e., the orbit Euler angles with respect to Earth's equatorial plane. These are the longitude  $\Omega$  of the orbit ascending node, the argument of pericenter  $\omega$ , and the orbit inclination  $i$ . These elements are not equally sensitive, in their secular and periodic effects, to the relativistic precessions. Anyway, the argument of pericenter  $\omega$  is sensitive to all of them, in particular to their secular effects.

In our analysis we determined the orbital elements of the two LAGEOS satellites and then we computed the residuals with the method explained in Ref. [22]. This is the method developed by one of us in 1996, and that has been always used in the LT effect measurements performed so far [23]. This point represents a crucial aspect in this kind of analysis, because we need a reliable way to obtain the residuals in the orbital elements which retain the original concept of *observed - computed* quantity, which usually refers to the tracking observable, the range for SLR data. With regard to the background gravity field, in our setup we included two different models: (1)

EGM96 and (2) EIGEN-GRACE02S. The gravity field plays a very significant role in this kind of measurement. The uncertainties in its harmonic coefficients, especially in the even zonal ones, are the major source of systematic effects, as we know very well in the case of the previous measurements of the LT effect. The EGM96 model [24], the current conventional model recommended by the International Earth Rotation Service (IERS) [19], is a multisatellite model derived over a time span of several years. The advantage of a multisatellite model resides in the different orbital characteristics of the satellites, such as different semimajor axis, eccentricity and inclination [25]. A clear disadvantage is represented by the fact that the tracking data and the quality of the orbit dynamical models are not homogeneous for all the satellites. EIGEN-GRACE02S [26] has been derived by GRACE mission, and has the characteristic to improve the gravity field knowledge with a limited amount of data, in particular in the medium and long wavelengths of its spectrum.

In the following we focus on the results obtained from the analysis of the satellite's pericenter shift, in particular, for LAGEOS II. Indeed, in the case of the pericenter the observable quantity is  $e\Delta\omega$ ; i.e., it depends on the satellite eccentricity  $e$ . Because LAGEOS II orbit is more eccentric, LAGEOS II is the best candidate for an accurate measurement of the total relativistic precession of the pericenter. In Table I the results expected for the relativistic precession rates in the pericenter are shown. By inspection, the total relativistic precession of

TABLE I. Rates in mas/yr of the secular relativistic precession on the argument of pericenter of the two LAGEOS satellites (1 mas/yr = 1 milli-arc-second per year).

| Rates                     | LAGEOS II | LAGEOS    |
|---------------------------|-----------|-----------|
| $\Delta\dot{\omega}^E$    | + 3351.95 | + 3278.77 |
| $\Delta\dot{\omega}^{LT}$ | - 57.00   | + 32.00   |
| $\Delta\dot{\omega}^{dS}$ | + 10.69   | - 5.99    |

the LAGEOS II pericenter is  $\Delta\dot{\omega}_{II}^{rel} \simeq 3305.64$  mas/yr.

The result of our 13 year analysis is shown in Figure 1 for the satellite argument of pericenter advance in the case of the EIGEN-GRACE02S model. The Fast Fourier Transform (FFT) of the residuals in the pericenter rate confirms the presence of the unmodeled YS effect in the integrated residuals of Figure 1. We fitted the residuals with a linear trend plus four periodic terms [27]. These terms come from the FFT and correspond to the following main spectral lines of the YS effect [28]:  $\dot{\lambda} + \dot{\omega}$  ( $\simeq 257$  days),  $\dot{\lambda} - \dot{\omega}$  ( $\simeq 624$  days),  $\dot{\Omega} + \dot{\lambda} + \dot{\omega}$  ( $\simeq 485$  days) and  $\dot{\Omega} - \dot{\lambda} + \dot{\omega}$  ( $\simeq 312$  days) [29]. For the linear trend slope we obtained a best value of  $\Delta\dot{\omega}_{II}^{meas} = 3306.58$  mas/yr, that corresponds to a fractional discrepancy of about 0.03% with respect to the prediction of GR. The result of the fit mainly depends

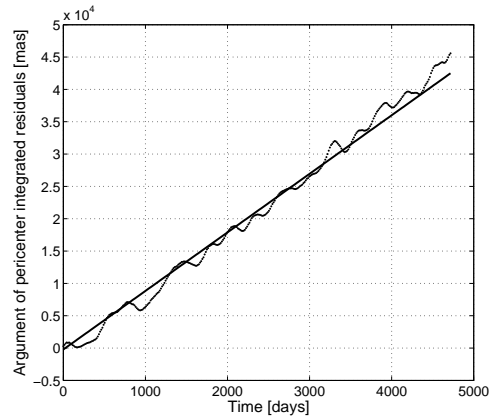


FIG. 1. Integrated residuals (dots) of LAGEOS II pericenter with the linear trend (continuous line) fitted together with four periodic terms with the periods of the main spectral lines of the YS effect. The fractional discrepancy between the slope of the linear trend and the prediction of general relativity is just  $2.8 \cdot 10^{-4}$ . The starting epoch is MJD 49004.

on the time span of the data analysis and on the number of periodic effects which are fitted together with the linear term. The worst result we obtained, by changing some of the initial conditions and the number of adjusted parameters, has been a 0.21% discrepancy between the value of the recovered slope and the predicted one, see Table II. Therefore, for our analysis of the LAGEOS

TABLE II. The fit results are mainly sensitive to the value of the intercept  $a$  of the fitting function. For  $a$  fixed to the value obtained from the residuals (280.9 mas), we obtained our best-fit result independently of the initial fixed value for the other parameters and the number of the periodic effects we added to the main four of the YS effect. Conversely, by varying the intercept up to  $\pm 50$  mas, value that corresponds to the rms of the range residuals of the adjusted state vector, we obtained our worst result.  $\Delta a$  is the variation for the intercept while  $\delta a$  and  $\delta b$  represent the adjustment of the given parameter.

| $\Delta a$ [mas] | $\delta a$ [mas] | $\delta b / \Delta\dot{\omega}_{II}^{rel}$ [%] |
|------------------|------------------|--|
| 0                | 0                | + 0.03   |
| $\pm 10$         | -9.9             | + 0.06   |
| $\pm 20$         | -19.9            | + 0.10   |
| $\pm 30$         | -29.9            | + 0.13   |
| $\pm 40$         | -39.9            | + 0.16   |
| $\pm 50$         | -49.9            | + 0.21   |

II pericenter general relativistic advance we assume the following conservative result:

$$\epsilon_{\omega} = 1 + (0.28 \pm 2.14) \cdot 10^{-3}. \quad (2)$$

The parameter  $\epsilon_{\omega}$  may be considered at the post-Newtonian level, and measures possible deviations from Einstein's GR, where  $\epsilon_{\omega} = 1$ . In the case of LAGEOS, we

obtained a worst best fit, with at least a 1% discrepancy with respect to the prediction of GR ( $\Delta\dot{\omega}_I^{rel} \simeq 3304.78$  mas/yr), due to the smaller eccentricity and the larger perturbations produced by the unmodeled YS effect [30].

The result obtained with the current analysis represents, to our knowledge, the most accurate measurement for the pericenter advance of a satellite orbiting the Earth ever made. In the PPN framework, it can be considered as a 0.03% measurement of the combination of the  $\gamma$  and  $\beta$  parameters. Indeed, since the leading contribution comes from Einstein's secular precession, we can consider  $\epsilon_\omega \simeq \epsilon_E = (2 + 2\gamma - \beta)/3$  [31]. The impact on the argument of pericenter of a possible NLRI described via a Yukawa-like interaction has been evaluated in [32], where also the contribution of the main systematic effects has been estimated. The secular effect is given by:

$$\Delta\dot{\omega}^{Yuk} \simeq 8.2923586 \cdot 10^{11} \alpha \text{ [mas/yr]} \quad (3)$$

and it corresponds to the peak value at a range  $\lambda = 6,081$  km, very close to 1 Earth radius. Hence, we can consider our measurement as an upper bound for the strength of a possible long-range interaction; we obtain:

$$|\alpha| \simeq |(1.0 \pm 8.9)| \cdot 10^{-12}. \quad (4)$$

This result represents a huge improvement in the constraint of the strength  $\alpha$  at 1 Earth radius. Previous results using Earth-LAGEOS and Lunar-LAGEOS measurements of  $GM$  where confined at the level of  $10^{-5}$  and  $10^{-8}$ . Our result for  $\alpha$  is comparable with those obtained with Lunar Laser Ranging (LLR) measurements at a characteristic scale of about 60 Earth radii, see Ref. [33]. With regard to the impact of the systematic errors on the measurement performed so far, these are mainly related with the uncertainty of the first even zonal harmonic  $J_2$  [32], and we can preliminarily assume a 2% value. In a forthcoming paper a full characterization of the systematic errors in the pericenter rate will be given, together with the results we obtained for the two LAGEOS satellites' ascending node longitude and inclination precessions.

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- $\epsilon_\omega$ . The best result has a 2% discrepancy with respect to the GR prediction in the case of LAGEOS II.
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